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An Analysis of Running Skyline Load Path

Ward W. Carson
Charles N. Mann

PACIFIC NORTHWEST FOREST AND RANGE EXPERIMENT STATION
U.S. DEPARTMENT OF AGRICULTURE
FOREST SERVICE
PORTLAND, OREGON

ABSTRACT

This paper is intended for those who wish to prepare an algorithm to determine the load path of a running skyline. The mathematics of a simplified approach to this running skyline design problem are presented. The approach employs assumptions which reduce the complexity of the problem to the point where it can be solved on desk-top computers of limited capacities. The results of this approach are compared with the exact catenary solution.

Keywords: Cable logging, mathematical analysis, logging, skidding (cableway), computer.

1.0 INTRODUCTION

Many logging system designers are now facing the problem of determining the load-carrying capability of the running skyline. There are basically three means for assessing this capability: a graphical-tabular handbook approach as presented by Lysons and Mann,[1/] [2/] a mathematical solution utilizing the large digital computing facilities,[3/] or a mathematical solution utilizing small desk-top computers.[4/]

Although each of these methods is directed to the problem of determining the load-carrying capability of the running skyline, there is a difference in approach. The procedures presented in references 1, 2, and 3 determine load-carrying capability under a given condition of deflection, whereas the procedure of reference 4 determines deflection, and ultimately a load path, for a specified load. The mathematical features of this latter approach to the problem are discussed in this paper.

2.0 MATHEMATICAL PROBLEM

To apply digital computers, either large facilities or small desk-top units, to skyline design problems, the skyline must be described mathematically. This description depends upon a clear statement of the mathematical problem, indicating the physical parameters and system characteristics that can be considered as known information. Such a statement of the running skyline problem can be made (refer to the cable-carriage configuration of fig. 1). The haulback, main, and grapple-opening lines are considered anchored at specific points, (a) and (b). These points are known to be separated horizontally by the span distance L and vertically by the elevation difference h. The carriage, located at (c), a known distance d from the anchor point (a), is supporting a known log load suspended free of the ground. The gross payload, W_G, is the sum of the carriage and log weights, and it is supported by the cable system at point (c). The operating tension, T_A, in the haulback at the left support is equal to the maximum allowable tension for this line, which is usually one-third of the breaking strength. The haulback line is a continuous line from the anchor point (a), running through the carriage and the block at (b), then back to the carriage where the other end is anchored. The weights of the main line, grapple-opening line, and haulback are specified. In the mathematical description, the main line and grapple-opening line are considered to function as one line, and the combined weight of both lines, w_3, is used. This approximation introduces negligible error and also makes the results applicable to running skyline cable configurations without grapple-opening lines or to those systems where the grapple-opening line remains slack and carries a negligible portion of the load.

[1/] Hilton H. Lysons and Charles N. Mann. Skyline tension and deflection handbook. Pac. Northwest Forest & Range Exp. Sta. USDA Forest Serv. Res. Pap. PNW-39, 41 p., illus., 1967.

[2/] Charles N. Mann. Mechanics of running skylines. Pac. Northwest Forest & Range Exp. Sta. USDA Forest Serv. Res. Pap. PNW-75, 11 p., illus., 1969.

[3/] Stanley K. Suddarth. Analysis of cable systems--the grapple rigged running skyline. Skyline Logging Symp. Proc., Oreg. State Univ., 1970.

[4/] Ward W. Carson, Donald D. Studier, and Hilton H. Lysons. Running skyline design with a desk-top computer/plotter. Pac. Northwest Forest & Range Exp. Sta. USDA Forest Serv. Res. Note PNW-153, 21 p., illus., 1971.

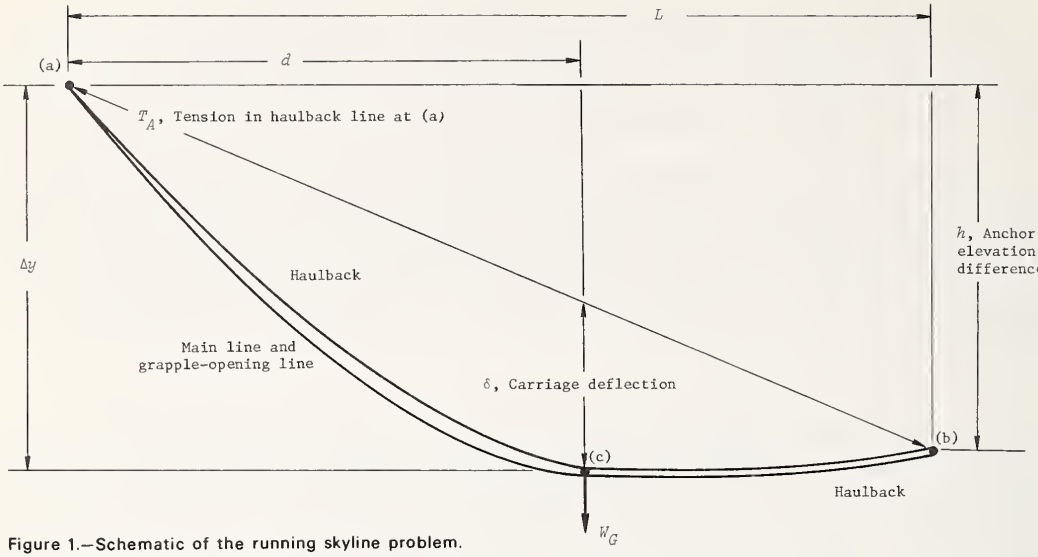


Figure 1.—Schematic of the running skyline problem.

The problem, as stated above, can be solved for the displacement, Δy, necessary to support the payload W_G at the point (c), while maintaining the operating tension T_A in the haulback line. When this problem is solved at a succession of points through the span of the skyline, a curve connecting these points is referred to as the load path of the carriage. This is the desired result of the analysis.

2.1 Problem Formulation

If the catenary description[5/] of this cable configuration was used in the problem formulation, a set of nonlinear, transcendental equations would result. A convenient solution of these would exceed the capacity of the desk-top type of computing facility. Since it is desired to provide a solution to the running skyline problem on a desk-top facility, another more simplified approach must be used.

The problem formulation employed in this paper consists of a force and moment balance approach, with simplifying assumptions about the weights and geometry of the cable segments. The problem is formulated as having known geometry, and load path is determined by an iterative technique designed around this formulation. The accuracy of the results is compared with results derived from a catenary formulation.

[5/] J. L. Meriam. Mechanics, part I - statics. New York, John Wiley & Sons, 340 p., illus., 1951.

2.2 Force Balance Formulation

The relationship between cable forces and geometry can be derived by considering the equilibrium of components of the running skyline system. The equations are written from force and moment balances for all line segments and for the skyline carriage (see fig. 2). For example, line segment 1 can be described by the vertical force balance on the line segment

$$V_1^a = R_1 + V_1^c ,$$

the horizontal force balance on the line segment

$$H_1 = \text{constant},$$

and the moment balance about the carriage

$$V_1^a = R_1 \frac{e_1}{d} + H_1 \frac{\Delta y}{d} .$$

In these expressions, R_1 represents the weight of the cable segment concentrated at a horizontal distance e_1 from the carriage, and the other forces and distances are described in figure 2.

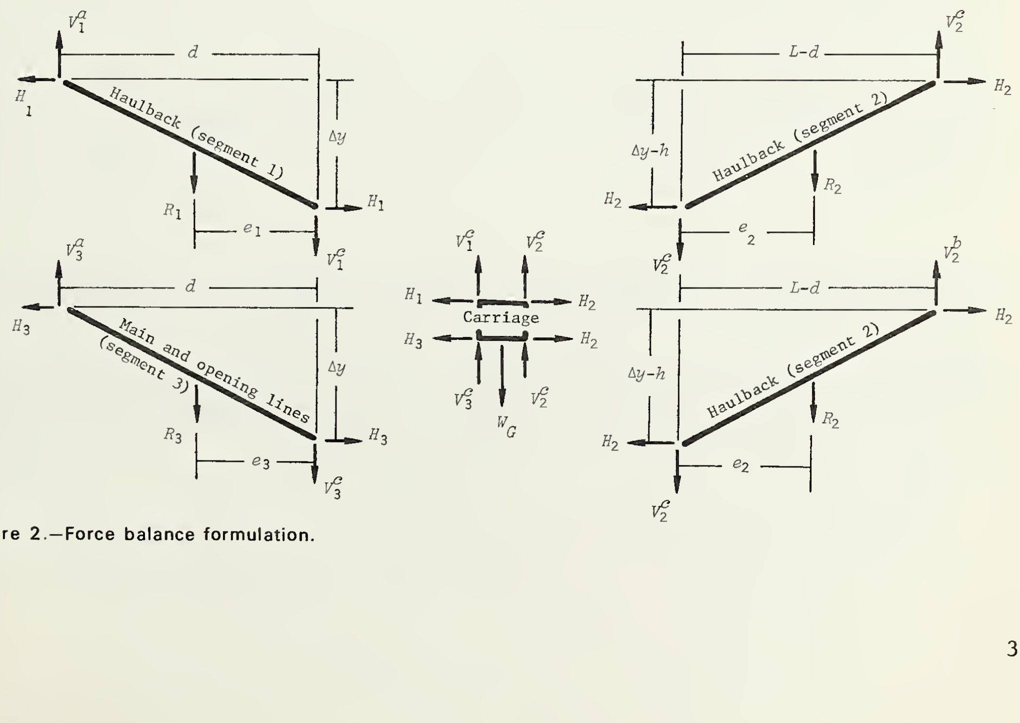


Figure 2.—Force balance formulation.

Line segments 2 and 3 can be described by similar expressions. These equations and the horizontal and vertical force balance equations on the carriage can be combined to provide direct solution for the horizontal tensions in each line segment. These expressions, listed in the order for solution with the necessary intermediate values, are

$$H_1 = \frac{-R_1 t_1 (t_2 - 1) + [(R_1 t_1 (t_2 - 1))^2 - (1+t_1^2)((R_1 (t_2 - 1))^2 - (T_1^c)^2)]^{1/2}}{(1+t_1^2)} ,$$

$$H_2 = \frac{-R_2 t_3 (t_4 - 1) + [(R_2 t_3 (t_4 - 1))^2 - (1+t_3^2)((R_2 (t_4 - 1))^2 - (T_2^c)^2)]^{1/2}}{(1+t_3^2)} ,$$

and

$$H_3 = 2H_2 - H_1 ,$$

where

$$t_1 = \Delta y/d ,$$

$$t_2 = e_1/d ,$$

$$t_3 = \Delta y\text{-}h/L\text{-}d ,$$

$$t_4 = e_2/L\text{-}d ,$$

T_1^c, tension in haulback segment 1 at carriage = $T_A - w_1 \Delta y$,

T_2^c, tension in haulback segment 2 at carriage = $T_A - w_1 \Delta y$,

and

w_1 is the weight per foot of the haulback.

The values of horizontal tension derived from these equations can be substituted in earlier expressions to determine the load-carrying capability of the skyline.

This formulation of the problem provides an approach for direct solution; however, the expressions presume knowledge of the line weights R and moment arms e of each line segment. Since these are determined by the weight distribution and the length and shape of each line segment, an iterative procedure is required for solution. However, with the approximations discussed in the next section, the iterations can be avoided and a direct solution is provided.

2.3 Force Balance Formulation With Straight-Line Approximations

As discussed in section 2.1, skylines are most accurately described as catenaries. With the geometry of the catenaries known, the weights R and moment arms e of each segment could be accurately determined and the equations of section 2.2 could be used for direct solution. The catenary lengths and shapes, however, are not known until solutions for tensions in each line have been established. Therefore, an exact solution would require iterations involving assumed tensions which provide catenary geometry, line weights, and moment arms, followed by computation of tensions to be compared to the values originally assumed.[6/] This procedure can be simplified by approximating the catenary geometry by straight-line segments between anchor points. The line segments would be envisioned as straight, rigid members with the uniform weight distribution of the cables and would be pinned at (a), (b), and (c). This assumption allows the line segment weights and moment arms to be expressed as

$$R_1 = w_1((d)^2 + (\Delta y)^2)^{1/2} \quad ,$$

$$e_1 = d/2 \quad ,$$

$$R_2 = w_2((L-d)^2 + (\Delta y-h)^2)^{1/2} \quad ,$$

$$e_2 = (L-d)/2 \quad ,$$

$$R_3 = w_3((d)^2 + (\Delta y)^2)^{1/2} \quad ,$$

and

$$e_3 = d/2 \quad .$$

If this assumption is made, the algebraic expressions of section 2.2 can be used to solve the problem directly. An iterative procedure is still required to establish the Δy which matches the gross payload of the skyline; however, the procedure is straightforward. The technique for determining Δy is discussed in the next section. The error of this approximation is presented in section 3.0.

2.4 Iteration For Δy

As discussed in the previous section, a good approximation to the running skyline problem can be formulated, based on known geometry. Since our problem is to determine geometry, specifically Δy, an iterative procedure based on successive solutions of the known geometry problem must be designed to determine the Δy which yields an appropriate gross payload W_G. This procedure, if designed for use with a desk-top computer, should require as little storage and as few instructions as possible. Such a procedure is suggested in this section.

[6/] Ward W. Carson and Charles N. Mann. A technique for the solution of skyline catenary equations. Pac. Northwest Forest & Range Exp. Sta. USDA Forest Serv. Res. Pap. PNW-110, 18 p., illus., 1970.

The iterative procedure determines Δy from the known relationship between Δy and W_G and a specified value of $(W_G)_{INPUT}$. The procedure was designed to insure convergence for all physically reasonable data and requires a minimum of instructions and storage in the process. The procedure depends upon an initial computation of W_G corresponding to the chord value of Δy. This will never be the correct value for Δy; however, it serves as a minimum in the iterations that follow.

A second $(\Delta y, W_G)$ pair is generated by choosing a δ corresponding to 1 percent of the skyline span and computing W_G. With this pair and the previous pair computed for the chord value of Δy, a linear relationship can be used to generate new estimates for Δy. The relationship, derived from the illustration in figure 3, is

$$(\Delta y)_I = \{(\Delta y)_{CALC} - (\Delta y)_{CHORD}\} \left\{ \frac{\left(W_G\right)_{INPUT} - \left(W_G\right)_{CHORD}}{\left(W_G\right)_{CALC} - \left(W_G\right)_{CHORD}} \right\} + (\Delta y)_{CHORD}$$

where

$(\Delta y)_{CALC}$ = the last value of Δy used in the calculations,

$(\Delta y)_{CHORD}$ = the value of Δy at the skyline chord,

$(W_G)_{INPUT}$ = the input value of gross payload,

$(W_G)_{CALC}$ = the last value of W_G computed,

$(W_G)_{CHORD}$ = the value of W_G computed with $(\Delta y)_{CHORD}$,

and

$(\Delta y)_I$ = an estimate for $(\Delta y)_{SOLN}$ which is used in the next computation of W_G.

This is the equation controlling the values of Δy used in the iterative procedure. For each pass through the calculations, the computed value of W_G is compared with $(W_G)_{INPUT}$. When these are within 1 percent of each other, the calculations are terminated and the value of Δy used to generate W_G is taken as the solution to the problem.

2.5 Numerical Example

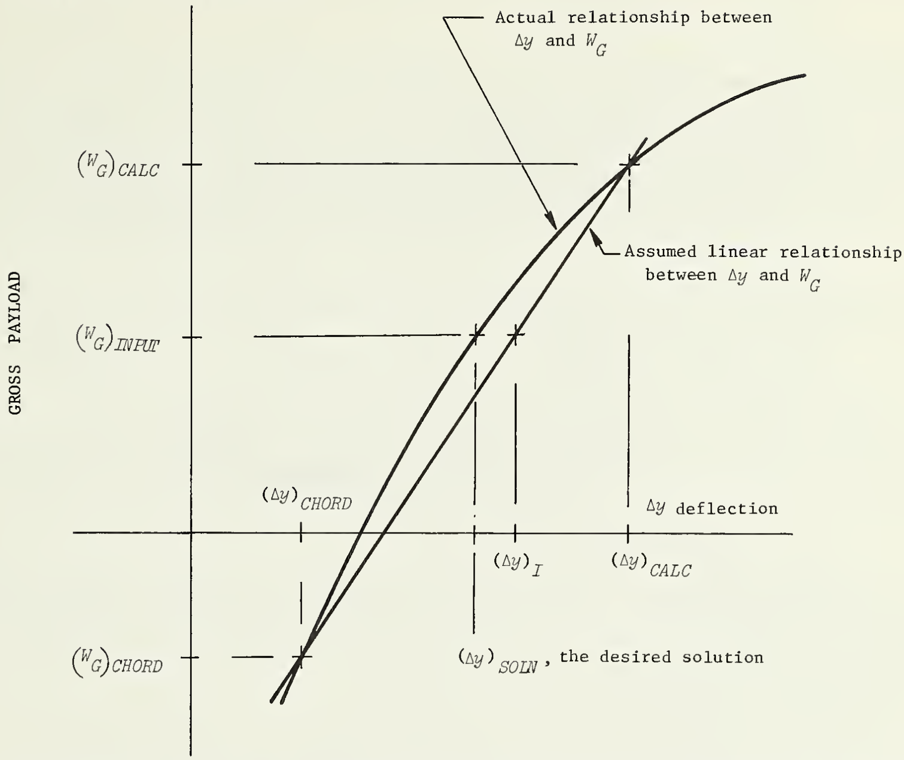
To demonstrate the procedure used to determine load-carrying capability, a numerical computation associated with the example of section 2.0 is discussed here. The geometry of figure 1 and the equipment specified provide the following input

L, skyline span = 800 feet,
h, elevation difference of anchors = 150 feet,
w_1, weight per foot of haulback = 1.42 pounds per foot,
w_3, weight per foot of main and grapple-opening lines combined = 2.84 pounds per foot,

and

T_A, operating tension of haulback = 26,500 pounds.

Figure 3.— Δy-W_G relationship.



For this example, the Δy for $(W_G)_{INPUT}$ = 25,000 pounds when the station location of d = 270 feet is determined.

The first step in the iterative procedure to determine the Δy is to establish $(\Delta y)_{CHORD}$ and $(W_G)_{CHORD}$. These are

$$(\Delta y)_{CHORD} = d\,\frac{h}{L} = 50.625$$

which leads to

$$(W_G)_{CHORD} = -1{,}350.837 \text{ pounds.}$$

The second estimate for Δy is taken as $(\Delta y)_{CHORD}$, plus 1 percent of the span. In this case,

$$(\Delta y)_2 = (\Delta y)_{CHORD} + L/100 = 58.625 \text{ feet.}$$

This gives a gross payload of

$$(W_G)_{CALC} = 970.412 \text{ pounds.}$$

With these two pairs of Δy, W_G, the equation of section 2.4 is used to predict the third estimate of Δy, namely,

$$(\Delta y)_3 = 141.441 \text{ feet.}$$

This gives a gross payload of

$$(W_G)_{CALC} = 25{,}290.21 \text{ pounds.}$$

Another application of the section 2.4 equation gives the final result

$$\Delta y = 140.45 \text{ feet}$$

and the corresponding gross payload of

$$W_G = 25{,}000.26 \text{ pounds}$$

which is close enough to $(W_G)_{INPUT}$ to be regarded as a solution.

Therefore, the solution for these conditions when the station d = 270.0 feet is

$$\Delta y = 140.45 \text{ feet.}$$

The four iterations required to arrive at this result are typical. This rapid convergence indicates that the linear relationship in section 2.4 is a good assumption.

3.0 ACCURACY OF THE STRAIGHT-LINE APPROXIMATION

The object of the analysis presented in the previous sections was to determine the load path geometry for specified skyline equipment, ground profile, and gross payload W_G. The accuracy of the approximation proposed in section 2.3 should then be discussed in terms of the geometric error of this solution compared to the results of the catenary formulation. This error can be defined as

$$E, \text{ percent error} = \frac{\delta_{FB} - \delta_{CAT}}{\delta_{CAT}} \text{ X } 100$$

where

δ_{FB} = the carriage deflection as computed by the force balance formulation containing the straight-line approximations

and

δ_{CAT} = the carriage deflection as computed by the catenary formulation.

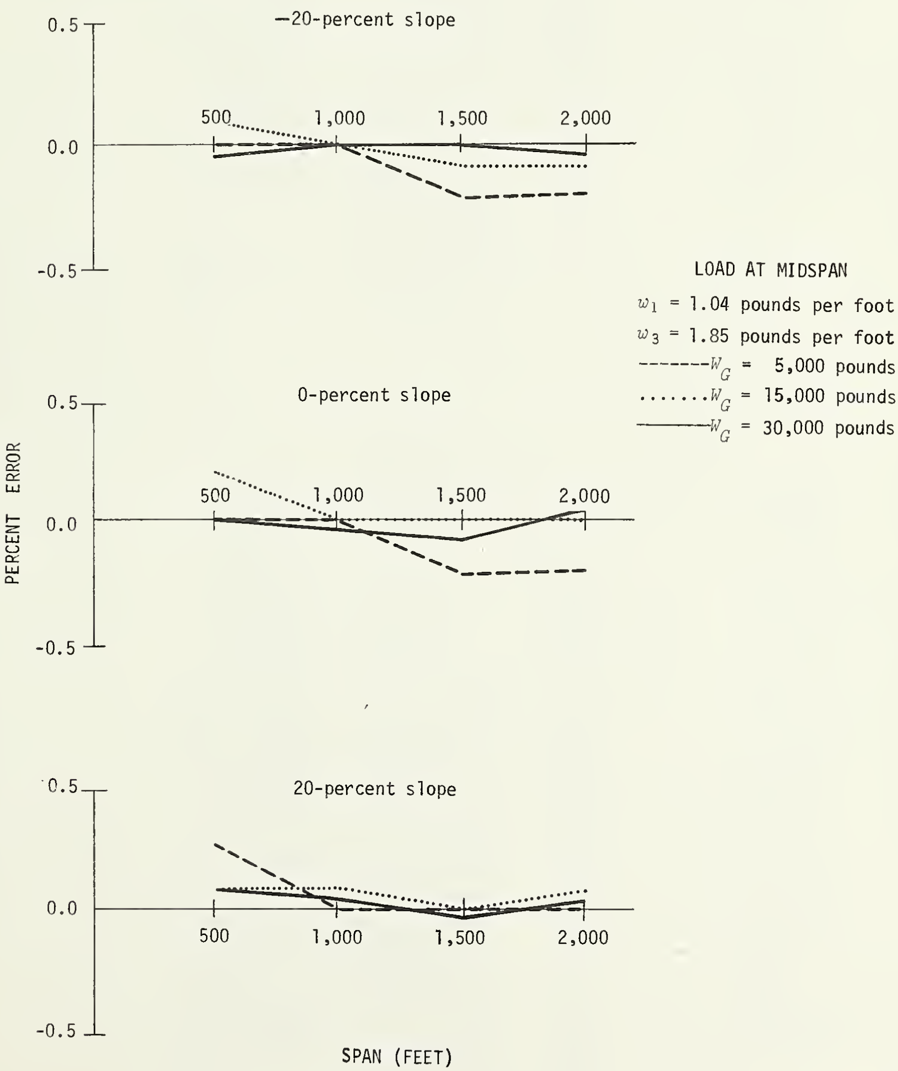
This error is influenced by the variables of span L, station d, slope S, line weights per foot w_1, w_2, and w_3, and gross payload W_G. Figure 4 displays the error for a range of these variables. Although a more thorough coverage of the variable ranges would be required for a full study of accuracy, the low errors

presented in these figures establish this solution as sufficiently accurate for practical applications of the running skyline system.

4.0 CONCLUSIONS

The approximate force balance solution to running skyline problems provides a method by which these problems can be solved on a desk-top computer with limited storage capability. Carson et al. (see footnote 4) gives the details of applying this solution with a typical desk-top computer/plotter system. Section 3 shows that this approximate solution is always within 1 or 2 percent of the exact catenary solution for the range of practical running skyline problems.

Figure 4.—Percent error in straight-line approximation.



Carson, Ward W., and Charles N. Mann.
1971. An analysis of running skyline load path. USDA Forest Serv. Res. Pap. PNW-120, 9 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

The mathematics of a simplified approach to running skyline problems is presented, and the simplified solution is compared with the exact catenary solution. This approach allows practical design of running skylines with desk-top computer/plotters.

Keywords: Cable logging, mathematical analysis, logging, skidding (cableway), computer.

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